

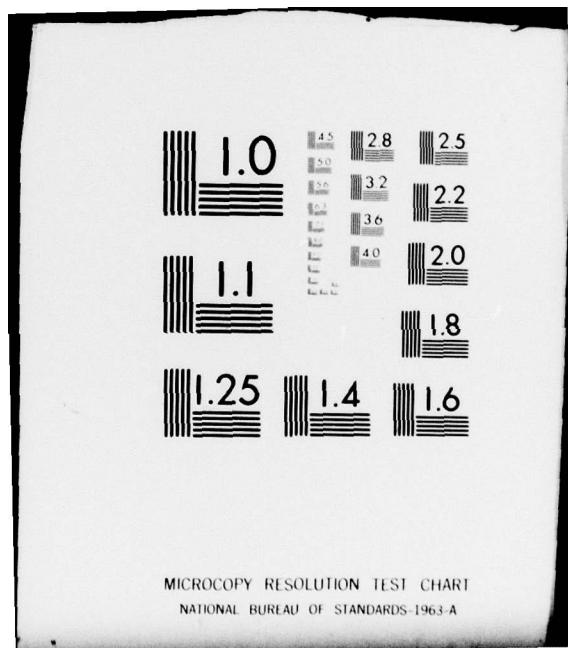
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SYNTHESIS OF OPTIMAL FILTERS USING MICROCOMPUTERS. (U)

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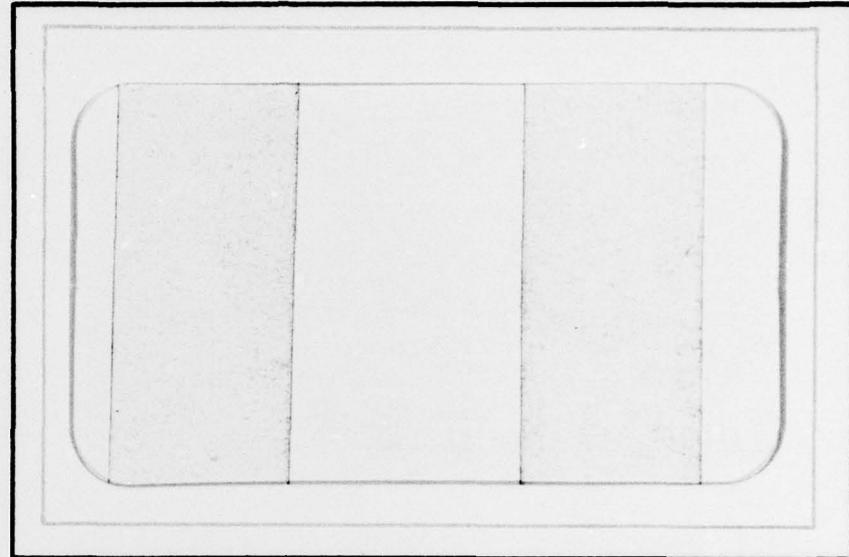


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DEPARTMENT OF THE NAVY  
UNITED STATES NAVAL ACADEMY  
ANNAPOLIS, MARYLAND 21402  
DIVISION OF ENGINEERING AND WEAPONS

Report EW-8-79

Synthesis of Optimal Filters  
Using Microcomputers

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August 1979

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## ABSTRACT

This project deals with optimal synthesis of an RC high-pass ladder network. It shows how to obtain a network with the maximum possible gain and minimum total series capacitance. The current transfer function, source and load conductances are specified. Lagrange multipliers method is used to obtain optimization. The entire process is completed using a microcomputer which can handle a large number of poles and can perform the optimization in a very short time.

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#### ACKNOWLEDGEMENTS

Although this project was carried out and completed in recent months, the idea was conceived several years ago while I was involved in a project funded by the U. S. Naval Academy Research Council (NARC). I would like to take this opportunity to express my appreciation.

Credit is due to Midshipman Richard A. Medley who spent many hours in his spare time to study the optimal synthesis and wrote the computer program.

Last, but not least, I would like to thank Joan Archer for typing the report.

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## I. INTRODUCTION

Due to our increasing dependence on integrated circuit technology, optimal synthesis of filters has become more and more important. The size of the chip is directly proportional to the amount of resistance and capacitance contained within. Thus, by minimizing these values, one can minimize the cost of each chip. This method, however, does not necessarily reduce the overall cost of production. The man-hours involved in arriving at these optimum values may more than offset the savings realized by smaller chips. This is especially true as the complexity of the filter increases. To alleviate this problem, one can use a microcomputer to aid in design. This reduction in man-hours makes optimal synthesis economically desirable.

The optimal synthesis of a double-terminated high-pass RC filter is very tedious if done by hand. The whole process, however, can be done in only a fraction of the time using a microcomputer which has some form of BASIC. The program is limited only by the storage capacity and accuracy of the microcomputer.

## II. SYNTHESIS PROCEDURES AND MICROCOMPUTER APPLICATION

To synthesize a double-terminated high-pass RC filter, as depicted in Fig. 1, one must first specify a high-pass current transfer function of the form:

$$I_{12}(s) = \frac{I_o}{I_s} = \frac{Hs^n}{(s + p_1)(s + p_2) \cdots (s + p_n)}$$

where the poles are simple and on the negative real axis. The source and load conductances must also be specified. From the current transfer function one can find the open-circuit impedance.

$$z_{12} = \frac{1}{g_\ell} I_{12}(s) = \frac{H}{g_\ell} \frac{s^n}{(s + p_1)(s + p_2) \cdots (s + p_n)}$$

This then can be expanded into partial fractions

$$z_{12} = \frac{H}{g_\ell} \left[ 1 + \frac{k_{12}^{(1)}}{s + p_1} + \frac{k_{12}^{(2)}}{s + p_2} + \cdots + \frac{k_{12}^{(n)}}{s + p_n} \right]$$

The residues of each pole can now be determined.

$$k_{12}^{(i)} = (s + p_i) \cdot \frac{z_{12} g_\ell}{H} \Big|_{s = -p_i} \quad i = 1, 2, 3, \dots, n$$

The calculation of the residues of  $z_{12}$  takes place in lines 350-430 of the example program shown in the Appendix, and are contained in the list A(I).

If the network is compact then:

$$k_{11}^{(i)} \cdot k_{22}^{(i)} = |k_{12}^{(i)}|^2$$

where  $k_{11}^{(i)}$  are the residues of  $z_{11}$  and  $k_{22}^{(i)}$  are the residues of  $z_{22}$ .

$$\text{Let } |k_{12}^{(i)}| = \alpha_i, \quad k_{11}^{(i)} = x_i \text{ and } k_{22}^{(i)} = \frac{\alpha_i^2}{x_i}$$

From the previously defined quantities  $z_{11}$  has the form:

$$z_{11} = \frac{H}{g_\ell} \left[ 1 + \frac{x_1}{s + p_1} + \frac{x_2}{s + p_2} + \cdots + \frac{x_n}{s + p_n} \right]$$

Also  $z_{22}$  has the form:

$$z_{22} = \frac{H}{g_\ell} \left[ 1 + \frac{\alpha_1^2/x_1}{s + p_1} + \frac{\alpha_2^2/x_2}{s + p_2} + \cdots + \frac{\alpha_n^2/x_n}{s + p_n} \right]$$

These two equations must satisfy the following constraints.

$$z_{11} \Big|_{s=0} = \frac{1}{g_s}$$

$$z_{22} \Big|_{s=0} = \frac{1}{g_\ell}$$

By using these constraints and the Lagrange multipliers method, one can obtain the following quadratic equation:

$$x^2 + \frac{1}{\sum_{i=1}^n \frac{\alpha_i}{p_i}} \left[ 1 - \frac{g_\ell}{g_s} \right] x - \frac{g_\ell}{g_s} = 0$$

Once  $X$  is known, it can be used to obtain the residues of  $z_{11}$  since  $x_i = X\alpha_i$ . The value of  $X$  and the residues of  $z_{11}$  are calculated in lines 440-490 of the example program with C3 containing  $X$  and the  $X(I)$  containing the residues of  $z_{11}$ .

By letting  $s = 0$  and substituting the expression for  $X$  into  $z_{11}$ , one can obtain an equation for  $H$ .

$$H = \frac{g_\ell}{g_s} \frac{1}{1 + X \cdot \sum_{i=1}^n \frac{\alpha_i}{p_i}}$$

$H$  is evaluated in line 500.

Once the residues of  $z_{11}$  and  $H$  have been determined, one must return the partial fraction expansion of  $z_{11}$  to a single fraction before a continued fraction expansion can be performed. To do this requires that one be able to expand a polynomial from its factored form. Consider the function.

$$Y(s) = (s + p_1)(s + p_2) \cdots (s + p_n)$$

$$Y(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n$$

where the coefficients  $a_i$  must be determined. It can be

shown that the coefficients  $a_i$  are a sum of products following the form:

$$a_1 = p_1 + p_2 + \cdots + p_n$$

$$a_2 = p_1 p_2 + p_1 p_3 + \cdots + p_{n-1} p_n$$

$$a_3 = p_1 p_2 p_3 + p_1 p_2 p_4 + \cdots + p_{n-2} p_{n-1} p_n$$

⋮

$$a_n = p_1 \cdot p_2 \cdot p_3 \cdots p_n$$

In evaluating these coefficients one must ensure that the number of FOR-NEXT loops is independent of the number of poles to maintain program flexibility.

To do this, consider the following truth table:

$p_3$	$p_2$	$p_1$
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

A FOR-NEXT loop can be used to form the product of  $p_1$  and  $p_n$  for each row. When the pole is low, however, it is skipped and the counter is not incremented. The product is then summed into a list where the position is determined by the value of the counter when the product is completed. Thus  $F(1)$  corresponds to  $a_1$ ,  $F(2)$  corresponds to  $a_2$  and so forth. To generate the truth table one can use a sinusoid with the frequency depending on the particular column. When the sinusoid is negative, the pole is considered high. When it is positive the pole is low. Lines 520-810 perform the

function of combining the partial fraction expansion. Once  $z_{11}$  is recombined, one can use an algorithm to obtain a Cauer 2 expansion of the admittance function. The algorithm is shown below.

$$Y_{11} = \frac{a_{13}s^2 + a_{12}s + a_{11}}{a_{23}s^2 + a_{22}s + a_{21}}$$

$$a_{11} \quad a_{12} \quad a_{13}$$

$$a_{21} \quad a_{22} \quad a_{23}$$

$$a_{31} \quad a_{32}$$

$$a_{41} \quad a_{42}$$

$$a_{51}$$

$$a_{61}$$

where  $a_{31} = \frac{a_{21} \cdot a_{12} - a_{11} \cdot a_{22}}{a_{21}}$

$$a_{32} = \frac{a_{21} \cdot a_{13} - a_{11} \cdot a_{23}}{a_{21}}$$

⋮

The element values are given by:

$$e_1 = \frac{a_{11}}{a_{21}}$$

$$e_2 = \frac{a_{21}}{a_{31}}$$

⋮

Line 980-1020 perform the Cauer 2 continued fraction expansion using the array S(J,I).

Finally, the element values are printed in a convenient form along with the network gain.

### III. EXAMPLES

A. Consider the following current transfer function with a load conductance of 5 mhos and a source conductance of 1 mho.

$$\frac{I_o}{I_s} = \frac{Hs^3}{(s+1)(s+2)(s+3)}$$

Solving for  $z_{12}$  yields:

$$z_{12} = \frac{H}{5} \left[ 1 + \frac{-1}{s+1} + \frac{8}{s+2} + \frac{-27}{s+3} \right]$$

Knowing the residues of  $z_{12}$ , one can solve for X and H.

$$X^2 + \frac{(1-5)}{9} X - 5 = 0$$

$$X = 2.47$$

$$H = 5 \cdot \frac{1}{1 + 2.47 \cdot 9} = .2155$$

With X known one can now determine the residues of  $z_{11}$  and thus the expression for  $z_{11}$ .

$$z_{11} = .0431 \left[ 1 + \frac{1.234}{s+1} + \frac{19.75}{s+2} + \frac{33.34}{s+3} \right]$$

$$z_{11} = \frac{.0431s^3 + 2.598s^2 + 8.448s + 6}{s^3 + 6s^2 + 11s + 6}$$

$z_{11}$  can now be expanded into a continued fraction using an algorithm for Cauer 2 as shown in Table I.

TABLE I

	6	11	6	1
1	6	8.448	2.598	.0431
2.351	2.552	3.402	.9569	
5.694	.4482	.3475	.0431	
.3148	1.424	.7118		
11.53	.1235	.0431		
.5734	.2153			
5	.0431			

The completed circuit is shown in Figure 2.

The entire synthesis process can be done using the micro-computer program shown in the Appendix. One must enter 3 for the number of poles, 1 for the source conductance, 5 for the load conductance, and each of the poles 1, 2, 3 entered separately. From this data the program prints out the value for each circuit element and the gain of the circuit.

B. Consider the following current transfer function with a load and source conductance of 1 mho.

$$\frac{I_o}{I_s} = \frac{Hs^5}{(s + 1)(s + 4)(s + 9)(s + 16)(s + 25)}$$

Solving for  $z_{12}$

$$z_{12} = H \left[ 1 + \frac{-115.7 \times 10^{-6}}{s + 1} + \frac{.2709}{s + 4} + \frac{-13.18}{s + 9} + \frac{92.47}{s + 16} + \frac{-134.6}{s + 25} \right]$$

Knowing the residues of  $z_{12}$ , one can solve for X and H

$$x^2 + \frac{(1-1)}{12.7} x - 1 = 0$$

$$x = 1$$

$$H = 1 \cdot \frac{1}{1 + 12.7} = 7.303 \times 10^{-2}$$

With X known one can now determine the residues of  $z_{11}$  which in this case are the absolute value of the residues of  $z_{12}$ .

$$z_{11} = .07303 \left[ 1 + \frac{115.7 \times 10^{-6}}{s + 1} + \frac{.2709}{s + 4} + \frac{13.18}{s + 9} + \frac{92.47}{s + 16} + \frac{134.6}{s + 25} \right]$$

$$z_{11} = \frac{.07303s^5 + 21.58s^4 + 678.1s^3 + 6537s^2 + 20280s + 14400}{s^5 + 55s^4 + 1023s^3 + 7645s^2 + 21076s + 14400}$$

$y_{11}$  can now be expanded into a continued fraction using an algorithm for Cauer 2 as shown in Table II

TABLE II

1	14400	21076	7645	1023	55	1
18.11	14400	20281	6537	678.1	21.58	.0730
3.525	795.3	1108	344.9	33.42	.9269	
2.957	225.6	292.6	72.93	4.792	.0730	
2.322	76.32	87.85	16.53	.6695		
2.322	32.86	24.05	2.813	.0730		
1.027	23.00	10.00	.5000			
2.322	13.78	2.300	.0730			
2.957	4.660	.3304				
3.524	1.322	.0730				
18.11	.0730					
1	.0730					

The completed circuit is shown in Figure 3.

Once again, the process can be done using a microcomputer in a similar fashion for Example A.

#### IV. CONCLUSION

As evidenced by the preceding examples, the use of microcomputers greatly reduces the time required to optimally synthesise a filter. One is required to know only the source conductance, load conductance and each pole to obtain the circuit gain and the value of each circuit element. In addition to decreased time, the microcomputer also provides a greater degree of reliability over long-hand methods. This is especially true as the filter becomes more complex.

Reduced time and greater reliability makes the use of a microcomputer essential in the optimal design of filters.

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- [3] E. N. Protonotarios, "Optimal Transfer-Function Synthesis of RC Ladders - Lumped and Distributed," IEEE Transactions on Circuits and Systems, Vol. CAS-21, pp. 49-56, January 1974.
- [4] R. A. Stein and A. I. A. Salama, "Resistance and Capacitance Minimization in Low-Pass RC Ladder Networks," IEEE Transactions on Circuits and Systems, pp. 27-31, January 1975.

Appendix

```

100 PRINT "WOULD YOU LIKE A DESCRIPTION OF THE PROGRAM?"
110 PRINT "ENTER 1 FOR YES, 0 FOR NO"
120 INPUT V
130 IF V=0 THEN 220
140 DIM R(11),F(11),T(21)
150 DIM S(22,11)
160 PRINT "THIS PROGRAM SYNTHESIZES AN OPTIMAL
170 PRINT HIGH-PASS FILTER. FROM THE CURRENT
180 PRINT TRANSFER FUNCTION ONE MUST ENTER EACH
190 PRINT POLE, THE SOURCE CONDUCTANCE, AND THE
200 PRINT LOAD CONDUCTANCE. (THE POLES ARE
210 PRINT ENTERED AS POSITIVE NUMBERS.)"
220 LET E=C=K=0
230 LET B=1
240 LET P=3.141592645
250 PRINT "INPUT THE NUMBER OF POLES (1<N<10)"
260 INPUT N
270 PRINT "INPUT THE SOURCE CONDUCTANCE"
280 INPUT G1
290 PRINT "INPUT THE LOAD CONDUCTANCE"
300 INPUT G2
310 FOR I=1 TO N
320 PRINT "INPUT P";I
330 INPUT L(I)
340 NEXT I
342 REM THE FOLLOWING STEPS CALCULATE THE RESIDUES OF Z12 WHICH
344 REM ARE THEN USED TO CALCULATE THE RESIDUES OF Z11
350 FOR I=1 TO N
360 FOR J=1 TO N
370 IF J=1 THEN 390
380 LET B=B*L(I)/*(L(J)-L(I))
390 NEXT J
400 LET A(I)=ABS (B*L(I))
410 LET B=1
420 LET C=C+A(I)/L(I)
430 NEXT I
440 LET C1=(1-G2/G1)/C
450 LET C2=SQR(C1*C1+4*G2/G1)
460 LET C3=(C2-C1)/2
470 FOR I=1 TO N
480 LET X(I)=A(I)*C3
490 NEXT I
492 REM WITH THE RESIDUES OF Z11 KNOWN, ONE CAN CALCULATE THE GAIN H
500 LET H=G2/G1/(1+C3*C)
502 REM THE FOLLOWING STEPS ARE USED TO PUT THE PARTIAL FRACTION
504 REM EXPANSION OF Z11 BACK INTO ONE FRACTION
510 LET D=2^N
520 FOR J=1 TO D-1
530 FOR I=1 TO N

```

```
540 IF SIN(P*(J+.001)*2/2^I)>0 THEN 570
550 LET E=E+1
560 LET B=B*L(I)
570 NEXT I
580 LET F(E)=F(E)+B
590 LET E=0
600 LET B=1
610 NEXT J
620 LET D=D/2
630 FOR J=1 TO N
640 FOR I=1 TO N-1
650 I<>J THEN 670
660 LET K=K+1
670 LET K=K+1
680 LET M(I)=L(K)
690 NEXT I
700 LET K=0
710 FOR O=1 TO D-1
720 FOR I=1 TO N-1
730 IF SIN(P*(O+.001)*2/2^I) 0 THEN 760
740 LET E=E+1
750 LET B=B*M(I)
760 NEXT I
770 LET Q(J,E)=Q(J,E)+B*X(J)
780 LET E=0
790 LET B=1
800 NEXT O
810 NEXT J
820 LET R(1)=F(1)
830 FOR I=1 TO N
840 LET R(1)=R(1)+X(I)
850 NEXT I
860 FOR J=1 TO N
870 FOR I=1 TO N
880 LET R(J+1)=R(J+1)+Q(I,J)
890 NEXT I
900 LET R(J+1)=R(J+1)+F(J+1)
910 NEXT J
920 LET S(1,N+1)=1
930 LET S(2,N+1)=H/G2
940 FOR I=1 TO N
950 LET S(1,N+1-I)=F(I)
960 LET S(2,N+1-I)=R(I)*H/G2
970 NEXT I
972 REM FINALLY A CAUER 2 EXPANSION IS DONE
980 FOR J=1 TO 2*N
990 FOR I=1 TO N
1000 LET S(J+2,I)=(S(J+1,1)*S(J,I+1)-S(J,1)*S(J+1,I+1))/S(J+1,1)
1010 NEXT I
1020 NEXT J
1030 PRINT "THE ELEMENT VALUES ARE:"
1040 FOR I=1 TO 2*N+1
1050 LET T(I)=S(I,1)/S(I+1,1)
```

```
1060 LET U=2*INT(I/2)
1070 IF U=I THEN 1110
1080 LET K=K+1
1090 PRINT "G";K;"=";T(I)
1100 GO TO 1130
1110 LET E=E+1
1120 PRINT "C";E;1/T(I)
1130 NEXT I
1140 PRINT
1150 PRINT "THE GAIN OF THIS FILTER IS";H
1160 END
```

Example A

FILTER

WOULD YOU LIKE A DESCRIPTION OF THE PROGRAM?  
ENTER 1 FOR YES, 0 FOR NO  
? 1

THIS PROGRAM SYNTHESIZES AN OPTIMAL HIGH-PASS FILTER. FROM THE CURRENT TRANSFER FUNCTION ONE MUST ENTER EACH POLE, THE SOURCE CONDUCTANCE, AND THE LOAD CONDUCTANCE. (THE POLES ARE ENTERED AS POSITIVE NUMBERS.)

? 3

INPUT THE SOURCE CONDUCTANCE

? 1

INPUT THE LOAD CONDUCTANCE

? 5

INPUT P 1

? 1

INPUT P 2

? 2

INPUT P 3

? 3

THE ELEMENT VALUES ARE:

G 1 = 1.

C 1 = 0.425307

G 2 = 5.69406

C 2 = 3.17637

G 3 = 11.5297

C 3 = 1.74378

C 4 = 5.

THE GAIN OF THIS FILTER IS 0.215297

Example B

FILTER

WOULD YOU LIKE A DESCRIPTION OF THE PROGRAM?  
ENTER 1 FOR YES, 0 FOR NO

? 0  
INPUT THE NUMBER OF POLES (1<N<10)  
? 5  
INPUT THE SOURCE CONDUCTANCE  
? 1  
INPUT THE LOAD CONDUCTANCE  
? 1  
INPUT P 1  
? 1  
INPUT P 2  
? 4  
INPUT P 3  
? 9  
INPUT P 4  
? 16  
INPUT P 5  
? 25

THE ELEMENT VALUES ARE:

G 1 = 1.  
C 1 = 5.52264E-2  
G 2 = 3.52454  
C 2 = 0.338232  
G 3 = 2.3223  
C 3 = 0.973783  
G 4 = 2.32244  
C 4 = 0.338229  
G 5 = 3.52454  
C 5 = 5.52264E-2  
G 6 = 1.

THE GAIN OF THIS FILTER IS 7.30256E-2

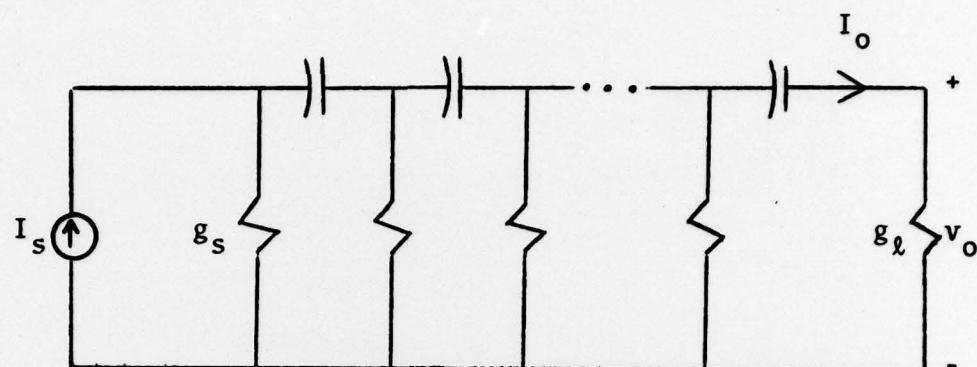


Figure 1

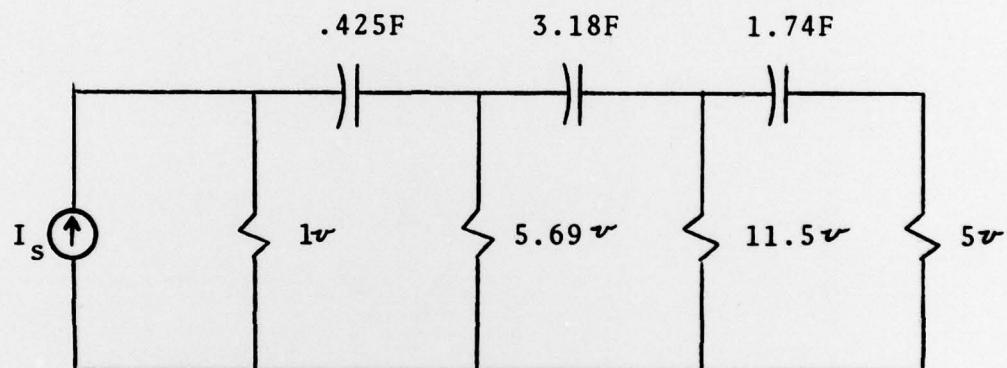


Figure 2

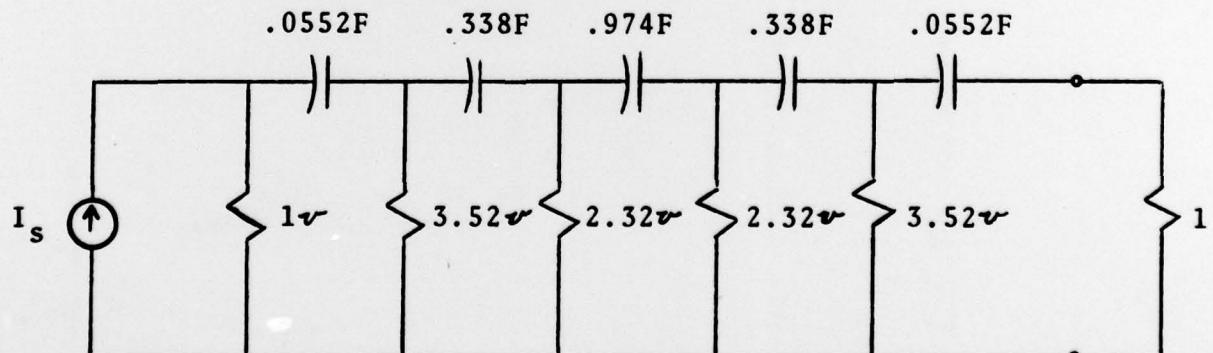


Figure 3

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